SOLUTIONS

2024-Mock JEE Advanced-3 (CBT) | Paper-2

PART-I	PHYSICS

SECTION-1

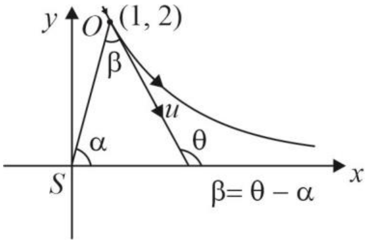
1.(AC) For the bead to remain stationary relative to wires without friction, the weight or bead and pseudo force on the bead should have resultant perpendicular to the wire. This is possible only for wire AC. Thus, wire AC is smooth.

The normal force applied by wire AB on the bead of mass m is: $N = \sqrt{(mg\cos\theta)^2 + (ma)^2}$

Thus, for this bead to not slip on wire, $\mu\sqrt{(mg\cos\theta)^2 + (ma)^2} \geq mg\sin\theta$

$\Rightarrow \quad \mu \geq \dfrac{g\sin\theta}{\sqrt{(g\cos\theta)^2 + (a)^2}}$

2.(AB) $y = \dfrac{2}{x}$; $\quad \dfrac{dy}{dx} = \dfrac{-2}{x^2}$



$\therefore \quad \tan\theta = -2$; $\quad \tan\alpha = 2$

$\tan\beta = \tan(\theta - \alpha) = \dfrac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}$

$\Rightarrow \quad \tan\beta = \dfrac{-2-2}{1-4} = \dfrac{-4}{-3} = \dfrac{4}{3}$

$\therefore \quad \cos\beta = \dfrac{3}{5}$; $\quad f_0 = f_s\dfrac{330 + u\cos\beta}{330}$

For u = 5 m/s, $f_0 = f_s \times \dfrac{333}{330} = \dfrac{111}{110}f_s$

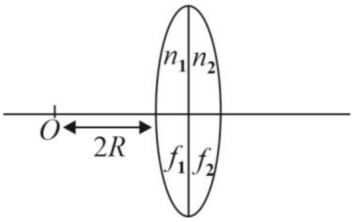
For u = 20 m/s, $f_0 = f_s \times \dfrac{342}{330} = \dfrac{114}{110}f_s$

3.(ABC) $\dfrac{1}{f_1} = \dfrac{1.5-1}{R} = \dfrac{1}{2R}$

$\dfrac{1}{f_2} = \dfrac{n_2 - 1}{R} \quad \Rightarrow \quad \dfrac{-\Delta f_2}{f_2} = \dfrac{\Delta n}{R}$

$\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

$-\dfrac{\Delta f}{f^2} = \dfrac{-\Delta f_2}{f_2^2} = \dfrac{\Delta n}{R} \quad \text{... (i)}$



$\dfrac{1}{V} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{-\Delta V}{V^2} = \dfrac{-\Delta f}{f^2} = \dfrac{\Delta n}{R} \quad \text{... (ii)}$

$\therefore \quad \Delta V = \frac{-\Delta n}{R} \times (2R)^2; \quad \Delta V = -4R\Delta n; \quad |\Delta V| = 4 \times 100 mm \times 1.5 \times 10^{-3}$ = 0.6 mm

$d\left(\frac{1}{F_1}\right) = 0$; since focal F_1 depends on n_1, that remains constant.

4.(BCD) $[M^{\circ} L^{\circ} T^1] = [L]^{\alpha}[L]^{\alpha}[ML^{-3}]^{\delta}[MLT^{-2}]^{\omega}$

$-2w = 1 \Rightarrow w = -\frac{1}{2}$

$\delta + w = 0 \Rightarrow \delta = -w = \frac{1}{2}$

$2\alpha - 3\delta + w = 0 \Rightarrow \alpha = 1$

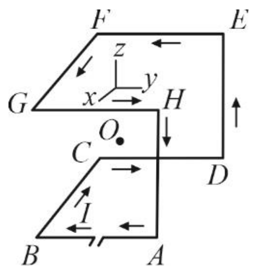
5.(ABD) ${}^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$

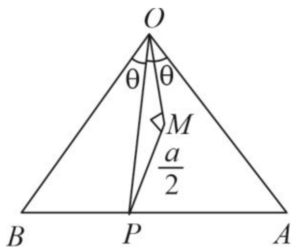
$p_1 = p_2$ (conservation of momentum)

$E = \frac{p^2}{2m}$

Hence, $E \propto \frac{1}{m}$ $\quad \therefore \quad \frac{E_1}{E_2} = \frac{m_2}{m_1} = \frac{234}{4} = \frac{117}{2}$

6.(BD) $OP = \frac{a}{\sqrt{2}}$





$BP = \frac{a}{2}; \quad OB = \frac{\sqrt{3}}{2}a$

$\sin\theta = \frac{BP}{OB} = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}; \quad B_1 = \frac{\mu_0 I}{4\pi \frac{a}{\sqrt{2}}}(2\sin\theta)$

$$= \frac{\mu_0 I}{4\pi a} 2\sqrt{2} \times \frac{1}{\sqrt{3}} = \frac{\mu_0 I}{\sqrt{6}\pi a} = B$$

$\vec{B}_1 = B_0\left(-\frac{1}{2}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}\right); \quad \vec{B}_2 = B_0\left(-\frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$

$\vec{B}_3 = B_0\left(\frac{1}{2}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}\right); \quad \vec{B}_4 = B_0\left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$

$\vec{B}_5 = B_0\left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\right); \quad \vec{B}_6 = B_0\left(\frac{1}{2}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right)$

$\vec{B}_7 = B_0\left(-\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\right); \quad \vec{B}_8 = B_0\left(-\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$

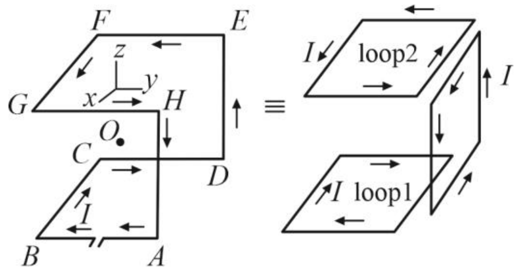
$\therefore \quad \vec{B}_{net} = B_0\left(0\hat{i} + \sqrt{2}\hat{j} + 0\hat{k}\right) \qquad = \frac{\mu_0 I}{\sqrt{6}\pi a} \times \frac{4}{\sqrt{2}}\hat{j} = \frac{2\mu_0 I}{\sqrt{3}\pi a}\hat{j}$

$\therefore \quad B_x = 0\,,\ B_y = \frac{2\mu_0 I}{\sqrt{3}\pi a}\,,\ B_z = 0$

Alternatively, $\vec{B}$ at the center due to loop 1 and loop 2 will cancel each other.

At the center due to loop 3 will be along + y-axis. $\vec{B}$

$\vec{B}_{net} = \frac{B_0}{\sqrt{2}} \times 4\hat{j} = \frac{2\mu_0 I}{\sqrt{3}\pi a}\hat{j}$



SECTION-2

7.(C)

8.(B) $\oint \vec{E}.dl = \pi r^2 \frac{\Delta B}{\Delta t} \qquad \Rightarrow \qquad E \times 2\pi r = \pi r^2 \frac{\Delta B}{\Delta t}$

$\Rightarrow \quad E = \frac{r}{2}\frac{\Delta B}{\Delta t}; \qquad qE = Ma_t = M\frac{\Delta V}{\Delta t}$

$\Rightarrow \quad q\frac{r}{2}\frac{\Delta B}{\Delta t} = \frac{M\Delta V}{\Delta t}$

$\Rightarrow \quad \Delta V = \frac{qr}{2M}\Delta B \qquad \ldots \text{(i)}$

$\mu = \pi r^2 I = \pi r^2 \frac{q}{2\pi r}V = \frac{qr}{2}V; \qquad \Delta\mu = \frac{qr}{2}\Delta V$

$= \frac{qr}{2} \times \frac{qr}{2M}\Delta B \qquad = \frac{q^2 r^2}{4M}\Delta B$

Hence, $\alpha = 4$

Initial state, $\quad T_0 + qV_0B_0 = \frac{mV_0^2}{r} \qquad \ldots \text{(ii)}$

Final state, $\quad T + qVB = \frac{MV^2}{r}$

$\Rightarrow \quad (T_0 + \Delta T) + q(V_0 + \Delta V)(B_0 + \Delta B) = \frac{M}{r}(V_0 + \Delta V)^2$

$\Rightarrow \quad (T_0 + qV_0B_0) + \Delta T + qB_0\Delta V + qV_0\Delta B = \left(\frac{MV_0^2}{r}\right) + \frac{2MV_0}{r}\Delta V$

$\Rightarrow \quad \Delta T + qV_0 \times \frac{2M}{qr}\Delta V + qB_0\Delta V = \frac{2MV_0}{r}\Delta V$

$\Rightarrow \quad \Delta T + qB_0\Delta V = 0 \qquad \Rightarrow \qquad |\Delta T| = qB_0\Delta V$

$\Delta\mu = \frac{qr}{2}\Delta V \qquad \therefore \qquad \frac{|\Delta T|}{\Delta\mu} = \frac{2B_0}{r}$

$\Rightarrow \quad |\Delta T| = \frac{2B_0\Delta\mu}{r} \qquad \therefore \qquad \beta = 2$

9.(B)

10.(D) Initially, $P_0V_0 = RT_0$ … (i)

Finaly $P = P_0 + \frac{kx_0}{A} = 2P_0$; $V = V_0 + Ax_0 = 2V_0$

$2P_0 \times 2V_0 = RT \quad \therefore \quad T' = 4T_0$

$\Delta U = nc_V \Delta T = 6RT_0$; $\Delta W = \oint PdV = \oint \left(P_0 + \frac{kx}{A}\right)(Adx)$

$$= P_0 A \int_0^{x_0} dx + K \int_0^{x_0} x\,dx = P_0 A \times x_0 + \frac{1}{2}kx_0^2$$

$$= Kx_0^2 + \frac{1}{2}Kx_0^2 \qquad = \frac{3}{2}Kx_0^2 = \frac{3}{2}(Kx_0)(x_0)$$

$$= \frac{3}{2}(P_0A)\left(\frac{V_0}{A}\right) = \frac{3}{2}P_0\,V_0 = 1.5RT_0 \qquad \therefore \qquad Q = \Delta U + \Delta W = 7.5RT$$

SECTION-3

1.(400) $Ah_2 = kt$

$\Rightarrow \quad h_2 = \frac{kt}{A}$ … (i)

$mg + 2\rho Ah_2 g = \rho Ah_1 g$

$\Rightarrow \quad m = \rho A(h_1 - 2h_2)$ ... (ii)

$t = t_1$, $m = \rho A(4h_2 - 2h_2)$

$m = 2\rho A\frac{kt_1}{A} \quad \Rightarrow \quad t_1 = \frac{m}{2\rho k} = \frac{400}{2\times1\times30}\times 60\,s$ = 400s

2.(800) $t = t_2$; $m = \rho A(3h_2 - 2h_2) \quad = \rho A\frac{kt_2}{A}$

$\Rightarrow \quad t_2 = \frac{m}{\rho k} = 800\,s$

3.(30) $I = mV_{CM} \quad \Rightarrow \quad V_{CM} = \frac{I}{m} = 2\,m/s$

(Torque about COM); $I \times \frac{l}{4} = \frac{ml^2}{12}\omega \quad \Rightarrow \quad \omega = \frac{3I}{ml}$ = 30 rad/s

4.(3.5) $K = \frac{1}{2}mV_{CM}^2 + \frac{1}{2}\frac{ml^2}{12}\omega^2 = \frac{I^2}{2m} + \frac{3I^2}{8m} = \frac{7}{8}\frac{I^2}{m}$ = 3.5 J

5.(400) S_1 is closed, S_2 is open.

$V_C = 220V$, $V_R = \sqrt{440^2 - 220^2} = 220\sqrt{3}V$

$\therefore \quad \frac{R_1}{X_C} = \frac{V_R}{V_C} = \sqrt{3} \quad \Rightarrow \quad R_1 = 100\sqrt{3}\,\Omega$

S_2 is closed, S_1 is open.

$V_C = 220\sqrt{3}V$, $V_R = \sqrt{440^2 - (220\sqrt{3})^2} = 220V$

$$\therefore \quad \frac{R_2}{X_C} = \frac{V_R}{V_C} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad R_2 = \frac{100}{\sqrt{3}}\Omega$$

S_1 and S_2 are closed. $\quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{100\sqrt{3}}{4}\Omega$

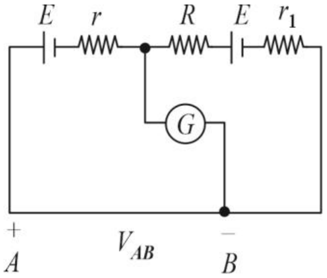
$$Z = \sqrt{R_{eq}{}^2 + X_C^2} = 100\sqrt{\left(\frac{\sqrt{3}}{4}\right)^2 + 1} = \frac{100}{4} \times \sqrt{19} = 25\sqrt{19}\ \Omega$$

$$V_0 = \frac{X_C}{Z} \times 440V = \frac{100}{25\sqrt{19}} \times 440 = \frac{4 \times 440}{4.4} = 400V$$

6.(4) $\quad I = \frac{440V}{Z} = \frac{440}{25\sqrt{19}} = \frac{440 \times 4}{100 \times 4.4} = 4A$

SECTION-4

7.(5)

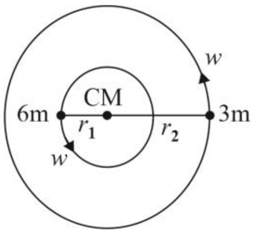


$$I = \frac{2E}{81 + r_1}; \quad V_{AB} = IR_0 \frac{l}{100} = \frac{2E}{81 + r_1} \times 60 \times \frac{70}{100} = \frac{84}{81 + r_1} E$$

$$V_{AB} = E - Ir \quad = E - \frac{2E}{81 + r_1} \times 1 \quad = E\left(1 - \frac{2}{81 + r_1}\right) \quad = E\left(\frac{79 + r_1}{81 + r_1}\right)$$

$$\therefore \quad \frac{84}{81 + r_1} E = E\frac{(79 + r_1)}{81 + r_1} \quad \Rightarrow \quad r_1 = 5\Omega$$

8.(81)



For $3M$: $\quad \frac{G\, 3M \times 6M}{R^2} = 3M \times W^2 \times \frac{6M}{9M} R$

$$\Rightarrow \quad W^2 = \frac{9GM}{R^3} \quad \ldots \text{(i)}$$

$$KE = \frac{1}{2} \times 3M \times W^2 r_1^2 + \frac{1}{2} 6M\, W^2 r_2^2$$

$$= \frac{1}{2} \times 3M \times \left(\frac{6M}{9M}\right)^2 R^2 W^2 + \frac{1}{2} \times 6M \times \left(\frac{3M}{9M}\right)^2 W^2 R^2$$

$$= \frac{1}{2} \times 3M \times 6M \times W^2 R^2 \left\{\frac{6M + 3M}{(9M)^2}\right\}$$

$$= \frac{1}{2} \times 3M \times 6M \times \frac{9GM}{R^3} \times R^2 \times \frac{1}{9M} \qquad = \frac{18}{2}\frac{GM^2}{R}$$

$$U = -G\frac{3M \times 6M}{R} = -\frac{18GM^2}{R}$$

$$E = K + U = -\frac{18}{2}\frac{GM^2}{R} = \frac{-9GM^2}{R} \qquad \therefore \qquad n = 81$$

9.(625) Electron emission will stop when the potential of the sphere grows equal to stopping potential.

$$\frac{hc}{\lambda} - \phi_0 = eV_0$$

$$\Rightarrow \qquad \frac{1240 \times 10^{-9}}{0.31 \times 10^{-6}} - 2.2 = V_0 \qquad \Rightarrow \qquad V_0 = 1.8V$$

Charge of the sphere, $Q = ne$

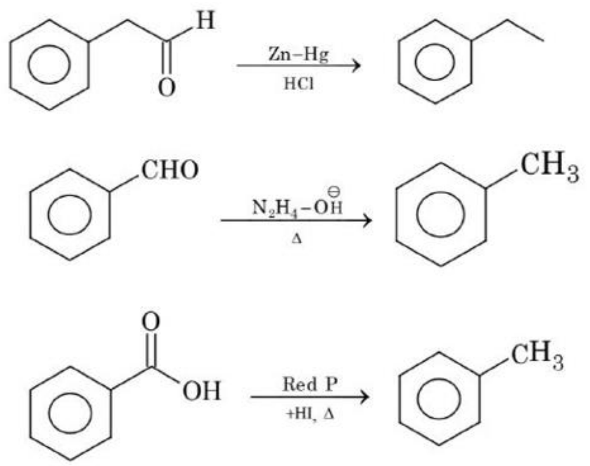
$$\therefore \qquad V_0 = \frac{KQ}{R} \qquad \Rightarrow \qquad 1.8 = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{5 \times 10^{-2}}$$

$n = 625 \times 10^5$ electrons

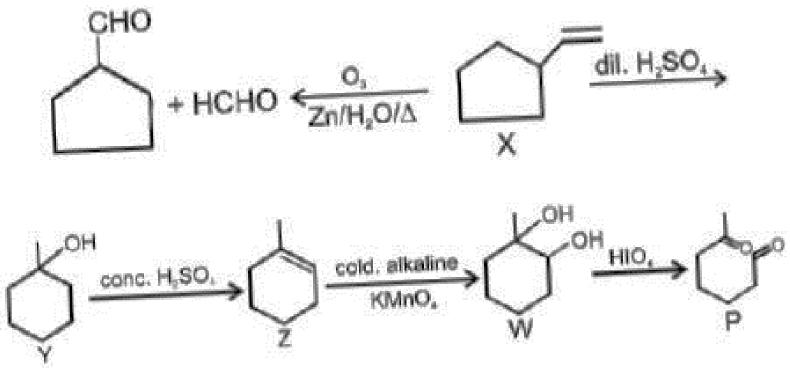
PART-II	CHEMISTRY

SECTION-1

1.(ABD)



2.(ABCD)



3.(BCD) $N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$

Initial mole	4	0	0
After diss. Mole	0	8	2

Mole ratio $= \frac{4}{10} = 2:5$

$t_{1/2} = \frac{0.693}{K} = \frac{0.693}{6.2\times10^{-4}} = 1117.7\,\text{sec}$

But it depends upon temperature, as K depends upon T

$t_{40\%} = \frac{2.303}{6.2\times10^{-4}}\log\frac{100}{(100-40)} = 824$ sec

Rate $r_1 = K[N_2O_5]$; if V is doubled then concentration becomes half.

$\therefore\ r_2 = K\times\frac{1}{2}[N_2O_5] \quad \therefore\ \frac{r_1}{r_2} = \frac{2}{1}$

4.(ABC) Moles of electron involved $= \frac{0.25\times9.65\times3600}{96500} = 0.09$

Mass of Zn involved $= \frac{0.09}{2}\times65.4 = 2.943\,\text{gm}$

Mass of MnO_2 involved $= 0.09 \times 87 = 7.83\,gm$

Mass of NH_4^+ involved $= 0.09 \times 18 = 1.62\,gm$

5.(BCD) Refer to theory.

6.(BD) $$\begin{array}{c} \quad\;\; O \qquad\;\; O \\ \quad\;\; \| \qquad\;\; \| \\ HO-P-O-P-OH \\ \quad\;\; | \qquad\;\; | \\ \quad\;\; H \qquad\;\; H \end{array}$$

SECTION-2

7.(D) Bomb calorimeter gives ΔU. Heat released will be greater due to H-bonding.

8.(B) I Meq. of NaOH = 20 ; Meq. of $H_2SO_4 = 20$

II Meq. of NaOH = 10 ; Meq. of $H_2SO_4 = 10$

In I : $\Delta H = -\dfrac{13.7 \times 10^3 \times 20}{1000}$ = 274 cal taken by 300 mL

In II : $\Delta H = -\dfrac{13.7 \times 10^3 \times 10}{1000}$ = 137 cal taken by 150 mL

9.(B) Since X forms white ppt of AgCl hence it should be (B).

10.(A) Pale yellow ppt of AgBr is formed with (A) option.

SECTION-3

1-2. **1.(12.33)** **2.(1.78)**

$$\Lambda = \kappa \times \frac{1000}{C} = \frac{4.95 \times 10^{-5} \times 1000}{0.001028} = 48.15\,S\,cm^2\,mol^{-1}$$

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{48.15}{390.5} = 0.1233$$

$$K = \frac{C\alpha^2}{(1-\alpha)} = \frac{0.001028 \times (0.1233)^2}{(1-0.1233)} = 1.78 \times 10^{-5}$$

Since, $\alpha = x \times 10^{-2}$

$x = 12.33$

$K = y \times 10^{-5}$

$y = 1.78$

3-4. **3.(16)** **4.(14)**

Total moles of Mg $= \dfrac{6}{24} = 0.25$

Moles of H_2 at STP $= \dfrac{3.36}{22.4} = 0.15$

$Mg + \frac{1}{2}O_2 \longrightarrow MgO$

0.1 mol 0.1 mol

$Mg + H_2SO_4 \longrightarrow MgSO_4 + H_2 \uparrow$

0.15 mol 0.15 mol

$MgO + H_2SO_4 \longrightarrow MgSO_4 + H_2O$

0.1 mol 0.1 mol

$\left(W_{H_2SO_4}\right)_i = 100 \times 1.4 \times 0.3 = 42\,gm$

$\left(W_{H_2SO_4}\right)_f = 42 - 0.25 \times 98 = 17.5\,gm$

(a) % w/w of final H_2SO_4 solution $= \frac{17.5}{100 \times 1.25} \times 100 = 14\%$

(b) W_{O_2} used $= \frac{0.1}{2} \times 32 = 1.6\,gm$

X = 16

Y = 14

5-6. **5.(7)** **6.(121)**

$Cu^{+2} + e \rightarrow Cu^{+}$

$2I^{-} \rightarrow I_2 + 2e$

$2S_2O_3^{2-} \rightarrow S_4O_6^{2-} + 2e$

Meq. of $Cu^{2+} =$ Meq. of liberated I_2 = Meq. of $Na_2S_2O_3 = 12.12 \times 0.1 \times 1 = 1.212$

$\frac{w_{Cu^{2+}}}{63.6/1} \times 1000 = 1.212$

$w_{Cu} = w_{Cu^{2+}} = 0.077$ $(\because Cu \xrightarrow{H_2SO_4} CuSO_4)$

$\%Cu = \frac{0.077}{1.10} \times 100 = 7\%$

X = % Cu = 7%

Meq. of $Cu^{+2} = 121 \times 10^{-2}$ $\Rightarrow$ Y = 121

SECTION-4

7.(11) $PV = nRT$ at point c

$2P_0 \times 4V_0 = 1 \times RT_c \Rightarrow T_c = \frac{8P_0V_0}{R}$

At point a,

$\Rightarrow T_0 = \frac{P_0v_0}{R} \Rightarrow T_c = 8T_0$

At point B,

$T_B = \frac{4P_0V_0}{R} - 4T_0$

Change in internal energy $= nC_vdT$

For path a to b $= 1 \times \frac{3}{2}R[3T_0] = \frac{9}{2}RT_0$

For path b to c $= 1 \times \frac{3}{2} R \times 4T_0 = 6T_0 R$

Total change $= \frac{9}{2} RT_0 + 6RT_0 = \frac{21RT_0}{2} = 10.5RT_0$

8.(5) $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2 T_2}{m_1 T_1}}$,

So, $\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{20 \times 1000}{4 \times 200}} = 5$

9.(9) $3Cl_2 + 6NaOH \rightarrow 5NaCl + NaClO_3 + 3H_2O$

The oxidation number of Cl in NaCl is $-1 \rightarrow a$ and in $NaClO_3$ is $+5 \rightarrow b$.

$\Rightarrow \quad 2 \times 5 - 1 = 9$

PART-III	MATHEMATICS

SECTION-1

1.(ABCD)

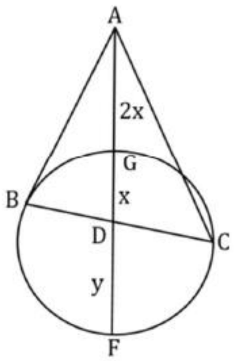
So, we have $n_1 = \binom{10}{3} = 120.$ Similarly, $n_2 = \binom{10}{3} + \binom{10}{2} = \binom{11}{3} = 165.$

And similarly, we have; $n_3 = \binom{13}{4} = 715$ and $n_4 = 2 \cdot \binom{10}{3} + \binom{10}{2} = 2.120 + 45 = 285.$

2.(AC) Let median through *A* meet *BC* at *D* and circle at *F*

Let $GD = x,\ DF = y$ then $AG \cdot AF = AB^2$

$2x(3x+y) = 36$ …(1)



$xy = 4$

$3x^2 + 4 = 18$

$x^2 = \frac{14}{3}$

So, $AD = 3x = \sqrt{42}$

Also, $AC^2 + AB^2 = 2(AD^2 + BD^2)$

$AC^2 + 36 = 2(42+4) = 2 \times 46 = 92$

$AC^2 = 56$

$AC = 2\sqrt{14}$

3.(CD) We have, $f(x) = x^4 - \frac{4}{3}x^3 + 2x^2 + px + q$

On differentiating with respect to *x*, we get

$f'(x) = 4x^3 - 4x^2 + 4x + p = 4x\left(x^2 - x + 1\right) + p$ As, $x^2 - x + 1 > 0 \forall x \in R,$

So, $4x\left(x^2 - x + 1\right) > 0 \forall\ x > 0$ and $4x\left(x^2 - x + 1\right) < 0\ \forall\ x < 0$

For $x \in (-1, \infty), 4x\left(x^2 - x + 1\right)$ can be positive or negative and have $f'(x)$ can be positive for some real values of *p* i.e., *f*(*x*) can be increasing for some real values of *p*.

For $x \in (0,1), 4x\left(x^2 - x + 1\right) > 0,$ so $f'(x)$

can be negative for infinite real values of 'p'.

$x > 0, p > 0 \Rightarrow f'(x) > 0$ and $x < 0, p < 0 \Rightarrow f'(x) < 0$

Domain of f is all real numbers, so $f'(x)$ can be positive or negative in this domain i.e., no real values of p & q can ensure monotonicity of f in its domain.

4.(AB) $2xy\dfrac{dy}{dx} = x^2 + y^2 + 1$

Put $y^2 = t,\ x\dfrac{dt}{dx} = x^2 + t + 1$

$$\frac{dt}{dx} - \frac{t}{x} = \frac{x^2+1}{x}$$

I.F. $= \dfrac{1}{x}$

Hence, $\dfrac{t}{x} = \displaystyle\int \frac{x^2+1}{x^2}dx = x - \frac{1}{x} + C$

$\therefore \quad y^2 = x^2 - 1, C = 0$ as $y(1) = 0$

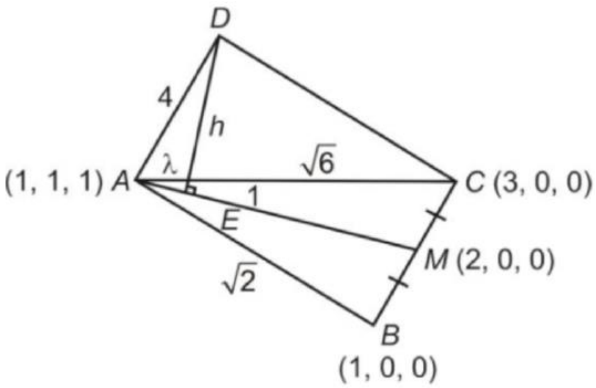
Now, $y(x_0) = \sqrt{3}$

$\Rightarrow \quad 3 = x_0^2 - 1$

$\Rightarrow \quad x_0^2 = 4$

$\therefore \quad x_0 = \pm 2$

5.(ACD)



Given $V = \dfrac{2\sqrt{2}}{3}$

Now, $\dfrac{1}{3}\cdot\dfrac{1}{2}\begin{vmatrix} \hat{i} & j & k \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix} h = \dfrac{2\sqrt{2}}{3} \Rightarrow h = 2 \Rightarrow$ (A) and (D)

Let E divides AM in the ratio $\lambda : 1$

Hence $E:\left(\frac{2\lambda+1}{\lambda+1},\frac{1}{\lambda+1},\frac{1}{\lambda+1}\right)$

Now, $(AE)^2+(DE)^2=(AD)^2$

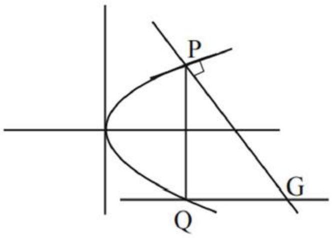
$$\left(\frac{2\lambda+1}{\lambda+1}-1\right)^2+\left(1-\frac{1}{\lambda+1}\right)^2+\left(1-\frac{1}{\lambda+1}\right)^2+4=16$$

$$\left(\frac{\lambda}{\lambda+1}\right)^2+2\left(\frac{\lambda}{\lambda+1}\right)^2=12$$

$$\Rightarrow \quad \left(\frac{\lambda}{\lambda+1}\right)^2=4 \quad \Rightarrow \quad \frac{\lambda}{\lambda+1}=2 \text{ or } -2$$

$\therefore$ These are two positions for E which are (–1, 3, 3) and (3, –1, –1).

6.(ABD)



Let $P(2t^2, 4t), Q(2t^2, -4t)$

Equation of normal at P $y+xt=4t+2t^3$ ...(i)

Equation of QG

$y=-4t$ …(ii)

Solving equation (i) and (ii) we get

$x=2t^2+8, \quad y=-4t$

Elimination t we get the locus of G as

$y^2=8x-64$

$y^2=8(x-8)$

SECTION-2

7.(B)

8.(D) Obviously $x^2+y^2\ge 0$ and $\Rightarrow$ $x+y\ge 0$ and is always an integer. Let r be any non-negative integer, and $x+y=r,$ then $[x^2+y^2]=r \Rightarrow r\le x^2+y^2<r+1$ which is a region lying between two concentric circles centred (0, 0) and having radii as $\sqrt{r}$ and $\sqrt{r+1}.$ Now, there will be an intersection between these lines and the given region between the circles, if $\sqrt{r}\le\frac{r}{\sqrt{2}}\le\sqrt{r+1}.$ So, r = 0, 2, 3 work. Summing all these chord lengths we get $4+\sqrt{6}-\sqrt{2}.$

9.(D)

10.(D) $f'(x)=2(x-x^2)e^{-x^2}$, which is increasing in (0, 1) & decreasing in $(1,\infty)$. Obviously $f(1)$ being its maximum value it's bounded from above and therefore it can't be surjective. But f is indeed differentiable.

Now we have $g(x)=\int_0^{x^2}\sqrt{t}e^{-t}dt.$

Substitute $\sqrt{t}=u \Rightarrow \dfrac{dt}{2\sqrt{t}}=du \Rightarrow dt=2udu$

$g(x)=\int_0^{x}2u^2e^{-u^2}du$

But we already have: $f(x)=2\int_0^{x}(t-t^2)e^{-t^2}dt$

So, we have:

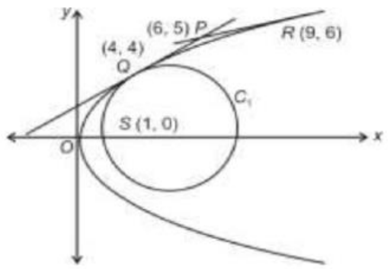
$f(x)+g(x)=\int_0^{x}2te^{-t^2}dt$

$\Rightarrow \quad f(x)+g(x)=(1-e^{-x^2})$

SECTION-3

1. (03.00)

2. (10.00)



Equation of tangent of slope m to $y^2=4x$ is

$y=mx+\dfrac{1}{m}$ …(1)

(i) As (1) passes through $P(6,5)$

$5=6m+\dfrac{1}{m}$

$\Rightarrow \quad 6m^2-5m+1=0 \quad \Rightarrow \quad m=\dfrac{1}{2}$ or $m=\dfrac{1}{3}$

Points of contact are $\left(\dfrac{1}{m_1^2},\dfrac{2}{m_1}\right)$ and $\left(\dfrac{1}{m_2^2},\dfrac{2}{m_2}\right)$

Hence $R(4, 4)$ and $Q(9, 6)$

Area of $\Delta PQR = \dfrac{1}{2}\begin{vmatrix}6 & 5 & 1\\4 & 4 & 1\\9 & 6 & 1\end{vmatrix}=\dfrac{1}{2}$

(ii) $y=\dfrac{1}{2}x+2 \Rightarrow x-2y+4=0$ …(2)

and $y=\dfrac{1}{3}x+3 \Rightarrow x-3y+9=0$

Now equation of circle C_2 touching $x-3y+9=0$ at (9,6), is

$(x-9)^2+(y-6)^2+\lambda(x-3y+9)=0$

As above circle passes through (1, 0), so

$64+36+10\lambda=0 \Rightarrow \lambda=-10$

Circle C_2 is

$x^2+y^2-28x+18y+27=0$ …(3)

Radius of C_2 is

$r_2^2=196+81-27=277-27=250$

$\Rightarrow \quad r_2=5\sqrt{10}$

3.(2)

4.(101)

Note that 0 and 1 obviously satisfy the equation E_1 Consider $f(t)=t^{x^2}+(10-t)^x$. Obviously, $f(5)=f(6)$ and therefore by Rolle's Theorem, there is some *c* such that $f'(c)=0$.

$\Rightarrow \quad x^2c^{x^2-1}=(10-c)^{x-1}$.

Because the exponents are positive therefore, *x* is positive.

If $x>1$, then $xc^{x^2-1}>c^{x^2-1}>c^{x-1}>(10-c)^{x-1}$ which is impossible since first and least term in this chain of inequalities are equal. Here we've used the fact that $c>5$.

If $0<x<1$, then similarly we can prove that $xc^{x-1}<(10-c)^{x-1}$. So a third solution doesn't exist. So the only solutions are 0, 1. For Question 10, using the idea of monotonicity we notice that the minima is attained at 50, 51. So their sum is 101.

5.(0)

6.(0)

Consider the integral $\int_0^1 \sqrt[3]{2x^3-3x^2-x+1}\,dx$ Say $x-\frac{1}{2}=t, dx=dt$ and

$\sqrt[3]{2x^3-3x^2-x+1}=\sqrt[3]{2x^3-3x^2-x+1}=\sqrt[3]{2t^3-\frac{5t}{2}}$

Which is an odd function in t¿ therefore we, have:

$\int_0^1 \sqrt[3]{2x^3-3x^2-x+1}\,dx=\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt[3]{2t^3-\frac{5t}{2}}dt=0$

Now the integral: $\int_0^1 \sqrt[3]{2x^3-3x^2-x+1}(2x-1)^2\,dx$ using the same transformation $x-\frac{1}{2}=t$ becomes:

$4\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt[3]{2t^3-\frac{5t}{2}}t^2dx=0$

SECTION-4

7.(8) There are four subset of {1, 2, 3,…., 9} that adds to greater than 21.

24 {7, 8, 9}, {6, 9, 8}23, {5, 8, 9} 22, {6, 7, 9}22

The number of 3 × 3 array having 7, 8, 9 as a row is $3(3!)(6!)$

This is true for each of the four sets.

Hence the number of 3×3 array having a row that sums > 21 is $(4)(3)(3!)(6!)$

Also total ways = $=9!$

$\therefore$ Probability $=\dfrac{(4)(3)(3!)(6!)}{9!}=\dfrac{1}{7}$

Note that exactly one row can contain elements whose sum is greater than 21.

8.(64) Consider $E:\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1(a>b)$ and $2ae=8$

$T:\dfrac{x}{a}\cos\theta++\dfrac{y}{b}\sin\theta=1 \quad \Rightarrow \quad A\left(\dfrac{a}{\cos\theta},0\right);B\left(0,\dfrac{b}{\sin\theta}\right)$

N: $\dfrac{ax}{\cos\theta}-\dfrac{by}{\sin\theta}=a^2-b^2$

$\Rightarrow \quad A'\left(\dfrac{a^2-b^2}{a}\cdot\cos\theta,0\right);B'\left(0,\dfrac{-\left(a^2-b^2\right)}{b}\sin\theta\right)$

$\Delta_1=\dfrac{ab}{2\sin\theta\cdot\cos\theta}\ and\ \Delta_2=\dfrac{(a^2-b^2)^2}{2ab}\sin\theta\cdot\cos\theta$

$\Delta_1\cdot\Delta_2=\dfrac{(a^2-b^2)^2}{4}=\dfrac{(ae)^4}{4}=64$

9.(0) Let $l=\displaystyle\int_0^{\pi/4}\dfrac{xdx}{\cos x(\cos x+\sin x)}$ $\quad\ldots(1)$

$l=\displaystyle\int_0^{\pi/4}\dfrac{\left(\dfrac{\pi}{4}-x\right)}{\left(\dfrac{1}{\sqrt{2}}\right)(\cos x+\sin x)\left(\dfrac{2}{\sqrt{2}}\cos x\right)}dx$ $\quad\ldots(2)$

$\therefore$ Adding (1) and (2), we get

$$2l=\frac{\pi}{4}\int_0^{\pi/4}\frac{dx}{\cos x(\cos x+\sin x)}=\frac{\pi}{4}\int_0^{\pi/4}\frac{\sec^2 xdx}{(1+\tan x)}=\frac{\pi}{4}\ln(1+\tan x)\Big]_0^{\pi/4}=\frac{\pi}{8}\ln 2$$

Let $\displaystyle\int_0^1\dfrac{\sin^{-1}x}{x}dx$

Put $\sin^{-1}x=t \Rightarrow x=\sin t\Rightarrow dx=\cos tdt$

$\therefore \quad l=\displaystyle\int_0^{\pi/2}\underset{(I)}{t}\cdot\underset{(II)}{\cot t}\,dt=(t\cdot\ln(\sin t))_0^{\pi/2}-\int_0^{\pi/2}\ln(\sin t)dt$

$0-\displaystyle\lim_{x\to0^+}tlin(\sin t)-\left(-\dfrac{\pi}{2}\ln 2\right)$

$\left(As\displaystyle\int_0^{\pi/2}\ln(\sin t)dt=-\dfrac{\pi}{2}\ln 2\right)=-\displaystyle\lim_{x\to0^+}\dfrac{\ln(\sin t)}{\dfrac{1}{t}}\left(\dfrac{\infty}{\infty}\right)+\dfrac{\pi}{2}\ln 2$

$$= -\lim_{x \to 0^+} \frac{\cot t}{\frac{-1}{t^2}} + \frac{\pi}{2} \ln 2 \;= \lim_{x \to 0^+} \left(\frac{t}{\tan t} \times t \right) + \frac{\pi}{2} \ln 2 = \frac{\pi}{2} \ln 2$$